

**REVIEW**  
**of the official reviewer for the dissertation work of**  
**Nauryz Targyn Atanbekovich on the topic "The method of heat polynomials and special functions for the problem of heat equation in regions with free boundaries and their application", submitted for the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 - Mathematical and computer modeling".**

№ p/p	Criteria	Compliance with the criteria (you must mark one of the answer options)	Justification of the official reviewer's position
1.	The topic of the dissertation (as of the date of its approval) corresponds to the directions of development of science and / or state programs	1.1 Compliance with priority areas for the development of science or government programs: <b>1) The dissertation was completed within the framework of a project or target program financed from the state budget (indicate the name and number of the project or program)</b> 2) The dissertation was completed within the framework of another state program (indicate the name of the program) <b>3) The dissertation corresponds to the priority direction of the development of science, approved by the Higher Scientific and Technical Commission under the Government of the Republic of Kazakhstan (indicate the direction)</b>	Applied research in mathematics and physics  Dissertation work's topic initially was related with AP05133919 "Heat polynomials and their applications to problems of heat and mass transfer", then was continued with supporting of the project AP09258948 "Problems with a free boundary in mathematical models of electrical contact phenomena"
2.	Importance for science	The work <b>makes</b> / does not make a significant contribution to science, and its importance is well <b>disclosed</b> / not disclosed	The results can be used to describe the heat transfer process in the electrical contact materials and have practical, theoretical meanings
3.	The principle of independence	Self-reliance level: 1) <b>High;</b> 2) Medium; 3) Low; 4) There is no independence	The mathematical modeling of the problems are constructed by scientific advisor but their analytical solutions, proofs of convergence, existence and uniqueness, graphical results by using program implementation obtained by author and it shows the level of independence in the performance of work is high
4.	The principle of	4.1 Rationale for the relevance of the dissertation:	The purpose of the dissertation is studying heat transfer

internal unity	<p>1) <b>Justified;</b>  2) Partially justified;  3) Not substantiated.</p>	<p>problems arising in electrical contact phenomena. The presented review of the special literature on this topic testifies to the author's knowledge of the achievements in this direction and the current state of the issue, which determined the topic of the dissertation. In the presented work, an alternative solution method is proposed, in which it is represented as a linear combination of heat polynomials and special functions with non-adjusted coefficients that satisfy the heat equation. The coefficients are chosen to ensure exactly or approximately the boundary conditions. In this dissertation, similarity type solution of the heat problem is considered and existence, uniqueness of the solution is proved and successfully discussed</p>
	<p>4.2 The content of the dissertation reflects the topic of the dissertation:  1) <b>reflects;</b>  2) partially reflects;  3) does not reflect</p>	<p>The content is fully consistent with the topic of the dissertation, basically the first and second parts are closely related, and the third part deals with problems with non-linear thermal coefficients where a special method is used called the similarity method.</p>
	<p>4.3. The purpose and objectives correspond to the topic of the dissertation:  1) <b>correspond;</b>  2) partially correspond;  3) do not correspond</p>	<p>The purpose and objectives correspond to the topic of the dissertation</p>
	<p>4.4 All sections and provisions of the dissertation are logically interconnected:  1) <b>are fully interconnected;</b>  2) the relationship is partial;  3) there is no relationship</p>	<p>All sections and provisions of the dissertation are devoted to the study of the dynamics of thermophysical processes in electrical contacts</p>
	<p>4.5 The new solutions proposed by the author (principles, methods) are argued and evaluated in comparison with the known solutions:  1) <b>there is a critical analysis;</b></p>	<p>In dissertation work, convergency of the linear combinations of the special functions, existence and uniqueness of the similarity type of solution of the nonlinear Stefan problem by using fixed point theory in</p>

		2) partial analysis; 3) the analysis is not one's own opinions, but quotes from other authors	Banach space are accurately verified
5.	The principle of scientific novelty	5.1 Are scientific results and provisions new? 1) <b>completely new</b> ; 2) partially new (25-75% are new); 3) not new (less than 25% are new)	Scientific results obtained from dissertation is new: - The analytical solution of the Stefan problem with generalized heat equation based on methods of special functions, Laguerre polynomials and congruent hypergeometric function and their convergency - Numerical approximate solution of the inverse spherical Stefan problem by using variational method - Exact solution of the spherical Stefan problem with nonlinear thermal coefficients by method similarity principle - Application of the similarity solution in blow-off explosion of closure electrical contacts
		5.2 Are the conclusions of the dissertation new? 1) completely new; 2) <b>partially new (25-75% are new)</b> ; 3) not new (less than 25% are new)	Conclusions of the dissertation according to the scientific results and dissertation provisions are partially new
		5.3 Technical, technological, economic or managerial decisions are new and justified: 1) completely new; 2) <b>partially new (25-75% are new)</b> ; 3) not new (less than 25% are new)	There are no managerial and economic decisions in the dissertation. The results of the dissertation can be used to characterize the thermophysical process for electrical materials
6.	Validity of the main conclusions	All key findings are <b>based</b> /not based on scientifically sound evidence, or reasonably well-founded (for qualitative research and arts and humanities courses)	All the main conclusions of the dissertation are confirmed by numerical results and substantiated. <i>But I would like to add one remark on the first general introduction to the integral error function, which is in the linear combinations of them</i> $u(x,t) = \sum_{n=0}^{\infty} [A_n u_n(x,t) + B_n u_n(-x,t)],$

			<p>where <math>u_n(\pm x, t) = t^{n/2} \operatorname{erfc}\left(\frac{\pm x}{2a\sqrt{t}}\right)</math>, why the second function has a negative variable if heat equation is defined on interval <math>0 &lt; x &lt; \alpha(t)</math>, the second term exits from this interval, it needs to be checked.</p>
7.	Basic provisions for defense	<p>The following questions need to be answered for each position separately:</p> <p>7.1 Is the position proven?</p> <p>1) <b>proven;</b>  2) rather proven;  3) rather unproven;  4) not proven</p> <p>7.2 Is it trivial?</p> <p>1) <b>yes;</b>  2) no</p> <p>7.3 Is it new?</p> <p>1) <b>yes;</b>  2) no</p> <p>7.4 Level to apply:</p> <p>1) narrow;  2) <b>medium;</b>  3) wide</p> <p>7.5 Is it proven in the article?</p> <p>1) <b>yes;</b>  2) no</p>	<p>Scientific position of the dissertation:</p> <ul style="list-style-type: none"> <li>- Solution of the inverse spherical Stefan problem with heat polynomials and variational numerical methods for material parameter AgCdO</li> <li>- Exact solution of the two-phase Stefan problem describing heat process in body with cross-section variable domain with special functions</li> <li>- Exact solution of the one-phase spherical Stefan problem with nonlinear thermal coefficients based on similarity transformation. Existence and uniqueness of the solution are proved</li> <li>- Application in mathematical modeling of blow-off instantaneous explosion of the electrical contact closure</li> </ul>
8.	The principle of certainty Reliability of sources and	<p>8.1 The choice of methodology is justified or the methodology is described in sufficient detail</p> <p>1) <b>yes;</b>  2) no</p>	<p>The methodology chosen in the dissertation work is substantiated and described in sufficient detail</p>


	information provided	8.2 The results of the dissertation work were obtained using modern methods of scientific research and methods of processing and interpreting data using computer technologies: 1) <b>yes</b> ; 2) no	Graphical representation of the solution of the problems are obtained by using program Mathcad
		8.3 Theoretical conclusions, models, identified relationships and patterns are proven and confirmed by experimental research (for areas of training in pedagogical sciences, the results are proven on the basis of a pedagogical experiment): 1) <b>yes</b> ; 2) no	Theoretical conclusions, models, identified relationships and regularities are proven and confirmed. <i>But it would be desirable to compare the results of the study with more other studies and show the advantages of the developed model</i>
		8.4 Important statements are <b>supported</b> / partially confirmed / not supported by references to relevant and reliable scientific literature	Important statements are confirmed by references to the current and reliable scientific literature
		8.5 Used literature sources are <b>sufficient</b> / not sufficient for a literature review	The sources of literature used are sufficient for literature review
9	Principle of practical value	9.1 The dissertation has a theoretical value: 1) <b>yes</b> ; 2) no	The results of the dissertation are theoretical in nature
		9.2 The dissertation is of practical importance and there is a high probability of applying the results obtained in practice: 1) <b>yes</b> ; 2) no	The practical significance of research lies in the methodology development of the new analytical and approximate method of solution which can be used in electrical contact system
		9.3 Are the practice suggestions new? 1) <b>completely new</b> ; 2) partially new (25-75% are new); 3) not new (less than 25% are new)	Practice offerings are completely new
10.	Quality of writing and design	Quality of academic writing: 1) high; 2) <b>medium</b> 3) below medium; 4) low.	The style of the dissertation is medium

**Conclusion:**

The dissertation work of Nauryz Targyn Atanbekovich on the topic "The method of heat polynomials and special functions for the problem of heat equation in regions with free boundaries and their application" submitted for the degree of Doctor of Philosophy (PhD) meets all the qualification requirements of the Rules for awarding degrees, presented for doctoral (PhD) dissertations. I consider Nauryz Targyn Atanbekovich to be awarded the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 – Mathematical and computer modeling".

**Official reviewer:**

Professor of the International Information  
Technology University,  
doctor of physical and mathematical sciences



(signature)



Rysbaiuly B.